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Discrete time transfer matrix method for dynamics of multibody system with real-time control

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ABSTRACT

By taking the control and feedback parameters into account in state vectors, defining new state vectors and deducing new transfer equations and transfer matrices for actuator, controlled element and feedback element, a new method named as the discrete time transfer matrix method for controlled multibody system (CMS) is developed to study dynamics of CMS with real-time control in this paper. This method does not need the global dynamics equations of system. It has the modeling flexibility, low order of system matrix, high computational efficiency, and is efficient for general CMS. Compared with the ordinary dynamics methods, the proposed method has more advantages for dynamics design and real-time control of a complex CMS. Adopting the PID adaptive controller and modal velocity feedback control on PZT actuators, and applying the proposed method and ordinary dynamics method, respectively, the tip trajectory tracking for a flexible manipulator is carried out. Formulations of the method as well as numerical simulation are given to validate the proposed method.

1. Introduction

With high-performance and high-precision demands for complex mechanical systems, such as lightweight robots, precision machinery, aircrafts, vehicles, and so on, the real-time, efficient dynamics prediction and control for a controlled multibody system (CMS) has become a research focus and difficulty in the filed of multibody system dynamics [1–3]. There are many theories and modeling methods for the dynamics of CMS. Roberson and Wittenburg (1966) developed R–W method [4] by introducing the graphics theory. Kane (1983) developed Kane equations [5] for studying on spacecraft dynamics by putting forward new concepts of partial velocity, partial angle velocity and generalized velocity. The Newton–Euler method, Lagrange method, variational principle and their derivative methods, such as Schiehlen method, Hamiton equations and so on are also widely used for CMS dynamics by many researchers, see Schiehlen [6,7], Pfeiffer [8]. When studying complex CMS dynamics using these above dynamics methods, it is necessary to develop global dynamics equations of system, the order of matrices involved is rather high increasing with the increase of freedom degrees of system, and the computational time is very huge. Sometimes, it is difficult to satisfy the demand for real-time control. Finding a new method to study CMS dynamics without global dynamics equations and improving computational efficiency of system dynamics has become more and more important.

The classical transfer matrix method (TMM) has been developed for a long time and has been used widely in structure mechanics and rotor dynamics of linear time invariant system. To linear system, Holzer [9] initially applied TMM to solve

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the problems of torsion vibrations of rods, Myklestad [10] applied TMM to determine the bending-torsion modes of beams, Thomson [11] applied TMM to more general vibration problems, Pestel [12] listed transfer matrices for elasto-mechanical elements up to 12th-order, Rubin [13] provided a general treatment for transfer matrices and their relation to other forms of frequency response matrices. Transfer matrices have been applied to a wide variety of engineering programs by many researchers, including Lin [14], Mercer [15], Mead [16], Henderson [17] and Murthy [18], dealing with beams, beam-type periodic structures, skin-stringer panels, rib-skin structures, curved multispan structures, cylindrical shells, stiffened rings, and so on. Dokanish [19] developed finite element-TMM to solve the problems of plate structure vibration analysis, by combining finite element method and TMM. Many researchers, such as, Ohga [20], Xue [21] and Loewy [22], studied and improved the finite element transfer matrix for structure dynamics. By using TMM, Li [23,24] investigated the problem of wave localization in disordered periodic multispan rib-stiffened plates and disordered periodic layered piezoelectric composite structures, respectively. Rui et al. [25] developed TMM of linear multibody system (MS-TMM) for vibrations analysis of linear multibody system by developing new transfer matrices and orthogonal property of multibody system. Kumar et al. [26] developed discrete time TMM for structure dynamics of time variant system. Rui et al. [27,28] developed discrete time TMM of multibody system (MS-DT-TMM) to study general multi-rigid-body system dynamics by combining and expanding the TMM and the numerical integration procedure. Rui et al. [29] extended MS-DT-TMM to study dynamics of multi-rigid–flexible-body system moving in plane. Compared with the ordinary dynamics methods, MS-DT-TMM has the advantages as follows: without global dynamics equations of system, low order of system matrix, high computational efficiency, and so on. These advantages of MS-DT-TMM provide a powerful tool for dynamics design of multibody system.

In this paper, by taking the control and feedback parameters into account in state vectors, defining new state vectors and deducing new transfer equations and transfer matrices for actuator, controlled element and feedback element, a new method named as the discrete time transfer matrix method for CMS (CMS-DT-TMM) is developed to study dynamics of CMS with real-time control. This method extends MS-DT-TMM and is effective for general CMS dynamics. Adopting PID adaptive controller and modal velocity feedback control on PZT actuators, applying the proposed method and ordinary dynamics method, respectively, the tip trajectory tracking of a flexible manipulator is carried out. The results gotten by the two methods have good agreements, which validate the proposed method.

This paper is organized as follows: In Section 2, the general theorems and steps of CMS-DT-TMM are developed. As an illustrative example, the dynamics model of a flexible manipulator and its control strategy are introduced in Sections 3 and 4, respectively. The transfer matrices of typical elements of the manipulator are developed in Section 5. The numerical simulation of this manipulator gotten by CMS-DT-TMM and by ordinary dynamics method is given to validate the method in Section 6. The conclusions and future works are presented in Section 7.

2. General theorems and steps of CMS-DT-TMM

According to the natural attribute of components, any CMS may be divided into a certain number of subsystems, which can be represented by various elements including bodies (rigid bodies, flexible bodies, lumped masses, and so on), hinges (linear springs, rotary springs, linear dampers, and so on) and actuators (PZT controller, impulse thruster, and so on). As shown in Fig. 1, the feedback output $\tilde{\mathbf{z}}_f$ of feedback element j is transformed to control input $\tilde{\mathbf{z}}_C$ of controlled element i by actuator, and $\tilde{\mathbf{z}}_{j\to i,R}$ is the reference input of actuator.

Considering the control strategies of system, the dynamics equations of any body and hinge can be developed. At the same time, the control equations of any actuator can be obtained. According to these dynamics equations and control equations, by defining new state vectors and combining with linearization methods, the dynamic properties and control laws of bodies, hinges and actuators can be expressed in matrices form. These matrices of elements are considered as building blocks, and when assembling them together according to structure of a CMS, the CMS dynamic can be obtained.

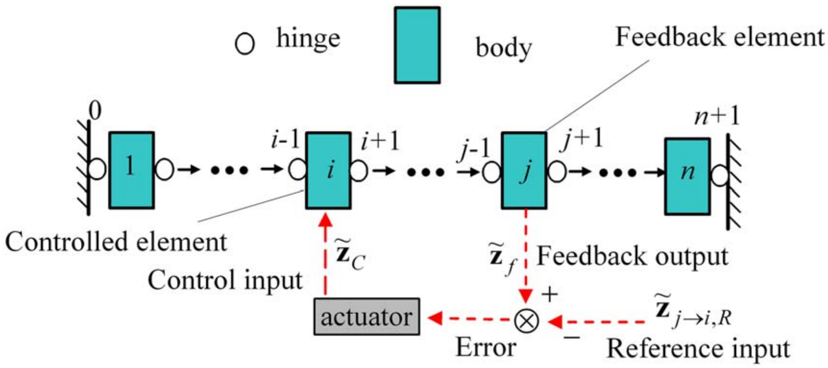

Fig. 1. Chain planer multibody system with real-time feedback.

In order to describe conveniently, the general theorems and steps of CMS-DT-TMM are introduced as follows by taking the chain planar CMS with real-time feedback shown in Fig. 1 for an example.

2.1. State vector

The state vector of the connection point among any rigid bodies and hinges moving in plane is defined, respectively, as

$$\mathbf{z} = [x, y, \theta, m, q_x, q_y, 1]^{\mathrm{T}} \tag{1}$$

The state vector of the connection point among any flexible bodies and hinges moving in plane is defined, respectively, as

$$\mathbf{z} = [x, y, \theta, m, q_x, q_y, q^1, q^2, \ldots, q^n, 1]^{\mathrm{T}} \tag{2}$$

The state vectors of the control input and feedback output are defined as

$$\tilde{\mathbf{z}}_C = [\tilde{u}_1^C, \tilde{u}_2^C, \ldots, \tilde{u}_g^C, 1]^{\mathrm{T}}, \tilde{\mathbf{z}}_f = [\tilde{u}_1^f, \tilde{u}_2^f, \ldots, \tilde{u}_p^f, 1]^{\mathrm{T}} \tag{3}$$

where x, y are the position coordinates of the connection point with respect to the inertial reference system. θ is the orientation angle of body-fixed reference system of involved body, m, q_x and q_y are the corresponding interior torque and interior forces in the same reference system, respectively. $q^1, \ldots, q^n$ are the generalized coordinates describing deformation of flexible bodies relative to the body-fixed reference system with modal method, the superscript 'n' is the highest order of the modal considered. $\tilde{u}_1^C, \ldots, \tilde{u}_g^C$ are the all control input parameters acting on the controlled element, $\tilde{u}_1^f, \ldots, \tilde{u}_p^f$ are the all feedback output parameters obtaining from the feedback element. The subscripts g, p denote the total number of the control and feedback parameters, respectively.

2.2. Linearization for dynamics equations and control equations of elements

For developing the transfer equations and transfer matrices of elements, some widely used linearization methods are simply introduced in this section; their detail can be seen in Ref. [28].

According to numerical integration procedures, the motion parameters $\ddot{\boldsymbol{\xi}}$ and $\dot{\boldsymbol{\xi}}$ of a CMS moving in plane at the time instant t_i are expressed as the linear function of $\boldsymbol{\xi}$ in form

$$\ddot{\boldsymbol{\xi}}(t_i) = \boldsymbol{\chi}_1 \boldsymbol{\xi}(t_i) + \boldsymbol{\chi}_{2,\xi}, \dot{\boldsymbol{\xi}}(t_i) = \boldsymbol{\chi}_3 \boldsymbol{\xi}(t_i) + \boldsymbol{\chi}_{4,\xi} \tag{4}$$

where the variable $\boldsymbol{\xi}$ may represent vector of the positions coordinates x, y or the orientation angle θ, respectively; $\dot{\boldsymbol{\xi}}$ and $\ddot{\boldsymbol{\xi}}$ represent the first order and the second order derivative of $\boldsymbol{\xi}$ with respect to time, that is, corresponding vectors of acceleration and velocity, or angular acceleration and angular velocity for planar motion, at the same time instant t_i. Using Newmark-β method [28], the quantities $\boldsymbol{\chi}_1, \boldsymbol{\chi}_{2,\xi}, \chi_3$, and $\boldsymbol{\chi}_{4,\xi}$ can be expressed as

$$\boldsymbol{\chi}_1 = \frac{1}{\beta \Delta T^2} \mathbf{I}_k, \boldsymbol{\chi}_{2,\xi} = -\frac{1}{\beta \Delta T^2} \left[\xi(t_{i-1}) + \Delta T \dot{\xi}(t_{i-1}) + \left(\frac{1}{2} - \beta \right) \Delta T^2 \ddot{\xi}(t_{i-1}) \right] \tag{5}$$

$$\boldsymbol{\chi}_3 = \gamma \chi_1 \Delta T, \boldsymbol{\chi}_{4,\xi} = \dot{\xi}(t_{i-1}) + \Delta T[(1-\gamma)\ddot{\xi}(t_{i-1}) + \gamma \boldsymbol{\chi}_{2,\xi}] \tag{6}$$

where $\Delta T = t_i - t_{i-1}$ is time step, β and γ are the coefficients of Newmark-β method. Bold capital symbol $\mathbf{I}_k$ is the unit matrix, its subscripts k denotes the order of the unit matrix.

The trigonometric functions at time t_i are expanded with respect to t_{i-1} using Taylor expansion, that is

$$\sin\theta(t_i) = \sin[\theta(t_{i-1}) + \Delta\theta] = \overline{s} + o(\Delta T^2), \cos\theta(t_i) = \cos[\theta(t_{i-1}) + \Delta\theta] = \overline{c} + o(\Delta T^2) \tag{7}$$

where

$$\overline{s} = \sin\theta(t_{i-1})\left\{1 - \tfrac{1}{2}[\dot{\theta}(t_{i-1})\Delta T]^2\right\} + \cos\theta(t_{i-1})[\dot{\theta}(t_{i-1})\Delta T + \tfrac{1}{2}\ddot{\theta}(t_{i-1})\Delta T^2]$$

$$\overline{c} = \cos\theta(t_{i-1})\left\{1 - \tfrac{1}{2}[\dot{\theta}(t_{i-1})\Delta T]^2\right\} - \sin\theta(t_{i-1})[\dot{\theta}(t_{i-1})\Delta T + \tfrac{1}{2}\ddot{\theta}(t_{i-1})\Delta T^2]$$

The trigonometric functions at time t_i can also be expressed as linear functions with respect to $\theta(t_i)$ using Taylor expansion, that is

$$\sin\theta(t_i) = \cos\theta(t_{i-1})\theta(t_i) + \overline{S} + o(\Delta T^2), \cos\theta(t_i) = -\sin\theta(t_{i-1})\theta(t_i) + \overline{C} + o(\Delta T^2) \tag{8}$$

where

$$\begin{cases} \overline{S} = \sin\theta(t_{i-1}) - \theta(t_{i-1})\cos\theta(t_{i-1}) - \dfrac{1}{2}\sin\theta(t_{i-1})[\dot{\theta}(t_{i-1})\Delta T]^2 \\ \overline{C} = \cos\theta(t_{i-1}) + \theta(t_{i-1})\sin\theta(t_{i-1}) - \dfrac{1}{2}\cos\theta(t_{i-1})[\dot{\theta}(t_{i-1})\Delta T]^2 \end{cases}$$

The multinomial in the dynamics equations can be approximated by

$$a(t_i)b(t_i) = a(t_{i-1})b(t_i) + a(t_i)b(t_{i-1}) - a(t_{i-1})b(t_{i-1}) + \dot{a}(t_{i-1})\dot{b}(t_{i-1})\Delta T^2 \tag{9}$$

The motion quantities $\xi(t_{i-1})$, $\dot{\xi}(t_{i-1})$, $\ddot{\xi}(t_{i-1})$ at the previous time instant are all known at time instant t_i. Thus, these quantities χ_1, $\chi_{2,\xi}$, χ_3, $\chi_{4,\xi}$, $\overline{s}$, $\overline{c}$ and so on are all definable for any subsystem for the time interval $(t_i - t_{i-1})$, and hence above formulations are valid.

2.3. Transfer equation and transfer matrix of element

Linearizing the dynamics equations and control equations using linearization methods as did in MS-DT-TMM [28,29], the single transfer equation of any element j (that is, any body, hinge and actuator) can be assembled as

$$\mathbf{z}_O = \mathbf{U}_j(t_i)\mathbf{z}_I \tag{10}$$

Here, $\mathbf{U}_j(t_i)$ is the transfer matrix of the jth element. It is the functions of motion quantities ($\xi(t_k)$, $\dot{\xi}(t_k)$ and $\ddot{\xi}(t_k)$, $k = i-1, i-2, \ldots$), control parameters and feedback parameters which are all known at time instant t_i. $\mathbf{z}_O$ and $\mathbf{z}_I$ are the state vectors of the input end and output end of the jth element, and they can be obtained easily by combing the definitions of Eqs. (1)–(3). For example, if the jth element is an actuator, then one can obtain $\mathbf{z}_I = \tilde{\mathbf{z}}_f, \mathbf{z}_O = \tilde{\mathbf{z}}_C$; if the jth element is the feedback element, then one can obtain $\mathbf{z}_I = \mathbf{z}_{j,j-1}$, $\mathbf{z}_O = [\mathbf{z}^{\mathrm{T}}_{j,j+1}, \tilde{\mathbf{z}}^{\mathrm{T}}_f]^{\mathrm{T}}$. $\mathbf{z}_{j,j-1}, \mathbf{z}_{j,j+1}$ are the state vectors of the connection point among feedback element j and hinges $j-1, j+1$.

In order to show the deduction process of the transfer equation and transfer matrix of element, a rigid body moving in plane without feedback and control parameters is taken as an example in follows.

As shown in Fig. 2, points I, O and C denote the inboard point, outboard point and mass center of rigid body, respectively; $O_2x_2y_2$ is the body-fixed reference system whose origin O_2 is on point I, oxy is the inertial reference system. (b_1, b_2) and (c_{c_1}, c_{c_2}) are the position coordinates of point O and mass center C with respect to the body-fixed reference system, respectively. So geometrical equations can be obtained

$$\theta_O = \theta_I = \theta \tag{11}$$

$$x_C = x_I + x_{\mathrm{IC}}, y_C = y_I + y_{\mathrm{IC}} \tag{12}$$

$$x_O = x_I + x_{\mathrm{IO}}, y_O = y_I + y_{\mathrm{IO}} \tag{13}$$

where (x_I, y_I) are the position coordinates of point I with respect to the inertial reference system. $x_{\mathrm{IC}} = c_{c_1}c_I - c_{c_2}s_I$, $y_{\mathrm{IC}} = c_{c_1}s_I + c_{c_2}c_I$, $x_{\mathrm{IO}} = b_1c_I - b_2s_I$, $y_{\mathrm{IO}} = b_1s_I + b_2c_I$, $s_I = \sin\theta_I$, $c_I = \cos\theta_I$.

The dynamics equations of the rigid body moving in plane without feedback and control can be obtained

$$m\ddot{x}_C = q_{x,I} - q_{x,O} + f_{x,C}, m\ddot{y}_C = q_{y,I} - q_{y,O} + f_{y,C} \tag{14}$$

$$J_I\ddot{\theta}_I + mx_{\mathrm{IC}}\ddot{y}_I - my_{\mathrm{IC}}\ddot{x}_I = m_O - m_I + m_C + q_{x,O}y_{\mathrm{IO}} - q_{y,O}x_{\mathrm{IO}} - f_{x,C}y_{\mathrm{IC}} + f_{y,C}x_{\mathrm{IC}} \tag{15}$$

where m is the mass of rigid body, (x_C, y_C) are position coordinates of mass center C with respect to inertial reference system, $q_{x,I}, q_{y,I}$ are internal forces acted on inboard point, $q_{x,O}, q_{y,O}$ are internal forces acted on outboard point, $f_{x,C}, f_{y,C}$ are external forces acted on mass center. J_I is the moment of inertia with respect to point I.

Linearizing Eq. (13) by Eq. (8), then one can obtain

$$x_O = x_I - y_{\mathrm{IO}}(t_{i-1})\theta_I + b_1\overline{C} - b_2\overline{S}, y_O = y_I + x_{\mathrm{IO}}(t_{i-1})\theta_I + b_1\overline{S} + b_2\overline{C} \tag{16}$$

Substituting Eq. (12) into Eq. (14), and linearizing by using Eqs. (4) and (8), then one can obtain

$$q_{x,O} = -m\chi_1x_I + m\chi_1y_{\mathrm{IC}}(t_{i-1})\theta_I + q_{x,I} + u_{57} \tag{17}$$

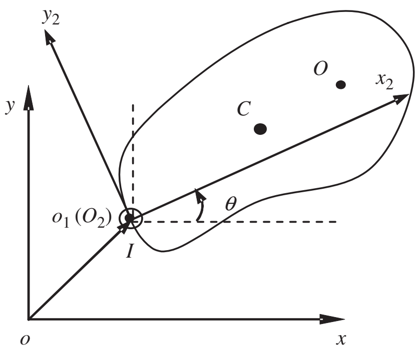

Fig. 2. Rigid body moving in plane without feedback and control.

$$q_{y,O} = -m\chi_1 y_I - m\chi_1 x_{IC}(t_{i-1})\theta_I + q_{y,I} + u_{67} \tag{18}$$

Substituting Eqs. (17) and (18) into Eq. (15), and linearizing by using Eq. (4), then one can obtain

$$m_O = u_{41}x_I + u_{42}y_I + u_{43}\theta_I + m_I + u_{45}q_{x,I} + u_{46}q_{y,I} + u_{47} \tag{19}$$

Combining Eqs. (11), (16)–(19), then the transfer equation of rigid body moving in plane without feedback and control can be obtained:

$$\mathbf{z}_O = \mathbf{U}\mathbf{z}_I \tag{20}$$

Transfer matrix

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & -y_{IO}(t_{i-1}) & 0 & 0 & 0 & b_1\overline{C} - b_2\overline{S} \\ 0 & 1 & x_{IO}(t_{i-1}) & 0 & 0 & 0 & b_1\overline{S} + b_2\overline{C} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ u_{41} & u_{42} & u_{43} & 1 & u_{45} & u_{46} & u_{47} \\ -m\chi_1 & 0 & m\chi_1 y_{IC}(t_{i-1}) & 0 & 1 & 0 & u_{57} \\ 0 & -m\chi_1 & -m\chi_1 x_{IC}(t_{i-1}) & 0 & 0 & 1 & u_{67} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

where the state vectors are defined as Eq. (1):

$$u_{41} = m\chi_1(y_{IO} - y_{IC}), u_{42} = m\chi_1(x_{IO} - x_{IC})$$

$$u_{43} = -m\chi_1 x_{IC}(t_{i-1})x_{IO} - m\chi_1 y_{IC}(t_{i-1})y_{IO} + J_I\chi_1, u_{45} = -y_{IO}, u_{46} = x_{IC}$$

$$u_{47} = -m_C + u_{67}x_{IO} - u_{57}y_{IO} + J_I\chi_{2,\theta} + (m\chi_{2,y_I} - f_{y,C})x_{IC} + (f_{x,C} - m\chi_{2,x_I})y_{IC}$$

$$u_{57} = f_{x,C} - m\chi_1(c_{c_1}\overline{C} - c_{c_2}\overline{S}) - m\chi_{2,x_C}, u_{67} = f_{y,C} - m\chi_1(c_{c_1}\overline{S} + c_{c_2}\overline{C}) - m\chi_{2,y_C}$$

$$x_{IC}(t_{i-1}) = (c_{c_1}c_I - c_{c_2}s_I)|t_{i-1}, y_{IC}(t_{i-1}) = (c_{c_1}s_I + c_{c_2}s_I)|_{t_{i-1}}$$

$$x_{IO}(t_{i-1}) = (b_1c_I - b_2s_I)|_{t_{i-1}}, y_{IO}(t_{i-1}) = (b_1s_I + b_2s_I)|_{t_{i-1}}$$

2.4. Overall transfer equation and overall transfer matrix of CMS

The overall transfer equation and overall transfer matrix of a CMS can be obtained by assembling the transfer equation and transfer matrix of each element. For the CMS shown in Fig. 1, applying Eq. (10) continuously, one can obtain

$$\mathbf{z}_{i,i-1} = \mathbf{U}_{i-1}\cdots\mathbf{U}_2\mathbf{U}_1\mathbf{z}_{1,0}, \tilde{\mathbf{z}}_C = \mathbf{U}^{\mathrm{fb}}_{j\to i}\tilde{\mathbf{z}}_f$$

$$\mathbf{z}_{j,j-1} = \mathbf{U}_{j-1}\cdots\mathbf{U}_{i+1}\mathbf{z}_{i,i+1}, \mathbf{z}_{i,i+1} = \mathbf{U}_i[\mathbf{z}^{\mathrm{T}}_{i,i-1}, \tilde{\mathbf{z}}^{\mathrm{T}}_C]^{\mathrm{T}}$$

$$[\mathbf{z}^{\mathrm{T}}_{j,j+1}, \tilde{\mathbf{z}}^{\mathrm{T}}_f]^{\mathrm{T}} = \mathbf{U}_j\mathbf{z}_{j,j-1}, \mathbf{z}_{n,n+1} = \mathbf{U}_n\mathbf{U}_{n-1}\cdots\mathbf{U}_{j+2}\mathbf{U}_{j+1}\mathbf{z}_{j,j+1} \tag{22}$$

where $\mathbf{U}_j$ $(j = 1, 2, \ldots, n)$ is the transfer matrix of the jth element, $\mathbf{U}^{\mathrm{fb}}_{j\to i}$ is the transfer matrix of actuator.

From Eq. (22), the overall transfer equation and overall transfer matrix of the CMS can be assembled and calculated as

$$\mathbf{U}_{\mathrm{all}}[\mathbf{z}^{\mathrm{T}}_{1,0}, \tilde{\mathbf{z}}^{\mathrm{T}}_C, \mathbf{z}^{\mathrm{T}}_{n,n+1}]^{\mathrm{T}} = 0 \tag{23}$$

where $\mathbf{U}_{\mathrm{all}}$ is the overall transfer matrix of the CMS.

2.5. Solutions of the controlled system dynamics

Once the overall transfer matrix of a CMS is known, the boundary conditions and reference input of the system can be applied and the unknown quantities in the boundary state vectors and all control parameters can be computed. Now, knowing the boundary state vectors and all control parameters completely, the state vectors and motion quantities of each element at time t_i can be computed reusing corresponding transfer equations of element similar to Eq. (10). Then the quantities of velocity, angular velocity, acceleration and angular acceleration at time t_i can be obtained using Eq. (4), respectively. Then entire procedure can be repeated for time t_{i+1} and so on. The corresponding flow chart of algorithms for this method is shown in Fig. 3.

It can be seen clearly that this method has following advantages: (1) the proposed method avoids global dynamics equations of a CMS, and simplifies its dynamics solving procedure; (2) irrespective of the size of a CMS, the matrices involved in the CMS-DT-TMM are always small, which greatly increases the computational speed and avoids the computing difficulties caused by too high matrix orders for complex CMS; (3) when using CMS-DT-TMM, the dynamics of CMS can be

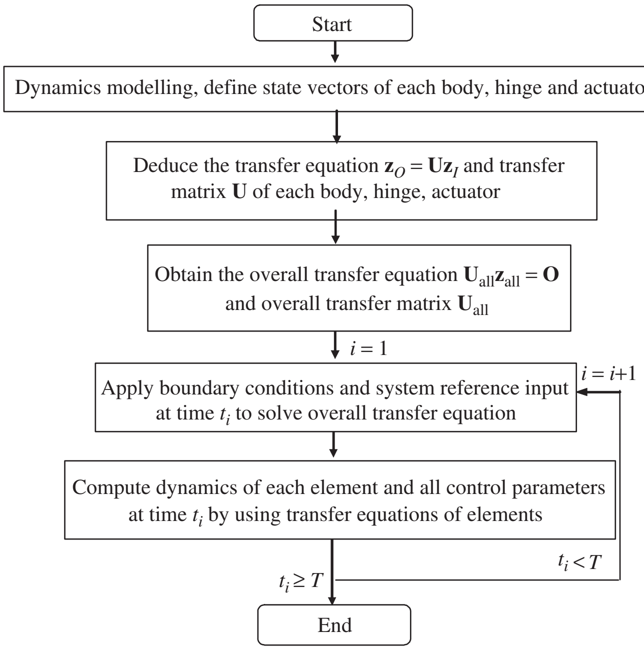

Fig. 3. Flow chart of algorithms for the proposed method.

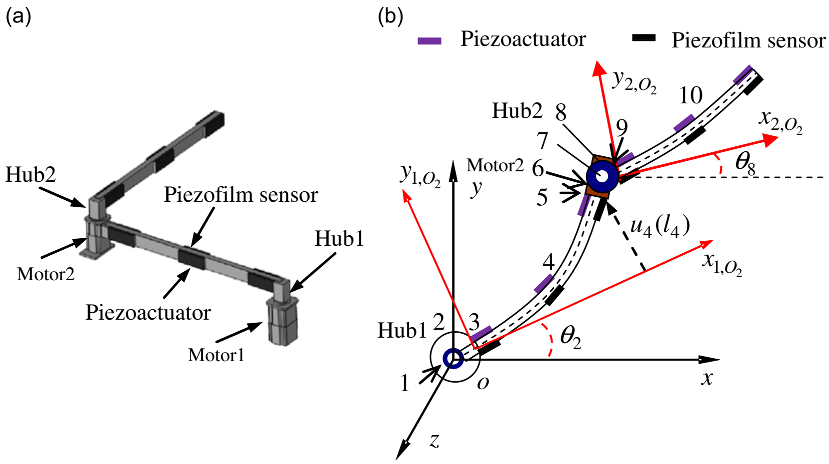

Fig. 4. Controlled planar flexible manipulator system and its dynamics model.

obtained only by solving the algebra equations. It avoids solving the differential equations or differential–algebra equations which are necessary when using ordinary dynamics methods, and simplifies the numerical arithmetic for CMS dynamics; and (4) this method provides flexibility in modeling complex CMS with varying configuration. That is, by creating a library of transfer matrices for commonly used elements and assembling these at the required locations, various configurations of the complex CMS can be modeled easily.

3. Dynamics model of controlled flexible manipulator system

A controlled planar flexible manipulator assembled by two flexible arms featuring surface-bonded PZT actuators and piezofilm sensors is shown in Fig. 4(a). Two servomotors are mounted at the each-link root. The position of each piezofilm sensor/PZT on corresponding arm is denoted by $[x^a_{i,1}, x^a_{i,2}]$ $(i = 1, 2, 3)$. The following assumptions are made: (1) the thickness of each actuator and sensor is thin adequately, and their effect on mass distribution and stiffness distribution of system is neglected. The arm is considered to be an Euler–Bernoulli beam and its axial deformation is neglected; (2) the polarization direction of each actuator and sensor is the same as the transverse vibration direction of corresponding arm; (3) each PZT actuator is perfectly bonded to arm and its voltage is uniform distributed; (4) each PZT actuator has constant thickness and the same width as the arm; and (5) the gravitational effect, friction and damping are neglected for simplicity.

The dynamics model of this controlled system can be obtained as shown in Fig. 4(b). Hub1, Motor2, and Hub2 are regarded as rigid bodies 2, 6 and 8, respectively. The arms regarded as Euler–Bernoulli beams 4 and 10, are connected with the corresponding Hubs by fixed hinges 3 and 9, respectively. The revolute hinges 1 and 7 connect Motors and their corresponding Hubs.

4. Control strategy

For precise tip trajectory tracking of the CMS shown in Fig. 4(a), a combined control strategy [30–35] is designed as follows: (1) considering each arm as rigid body and using inverse kinematics theory, the desired orientation angles $\theta_{2,d}, \theta_{8,d}$ of arms (2, 8) can be obtained according to the desired tip trajectory; (2) combining classical PID control with BP nerve network (BPNN), the PID adaptive controller is designed. The control torques τ_2', τ_8' of servomotors are designed by using these PID adaptive controllers, and the fast tracking of the orientation angles of each arm are realized; (3) based on modal velocity feedback, the command voltages $V_i(t)$ applied to PZT actuators can be obtained, and the active vibration control and precise tip trajectory tracking are realized. The control block diagram is shown in Fig. 5.

4.1. PID adaptive controller

As the PID adaptive controller [36] can realizes the adaptive modification of its controller parameters, the robustness and adaptive capability of system can be improved. In this paper, a three-layer BPNN as shown in Fig. 6 is adopted, and its output layer are relative to PID controller parameters K_p, K_i, K_d, respectively. The control equation for PID controller is

$$\tau'(t_i) = \tau'(t_{i-1}) + \Delta\tau'(t_i) \tag{24}$$

where

$$\Delta\tau'(t_i) = K_p[e(t_i) - e(t_{i-1})] + K_i e(t_i) + K_d[e(t_i) - 2e(t_{i-1}) + e(t_{i-2})] \tag{25}$$

$e = \theta - \theta_d$ is the tracking error of the orientation angle, θ and θ_d are the actual orientation angle and desired orientation angle, respectively.

Define the output of the input layer of BPNN as

$$O_j^{(1)} = x(j)(j = 1, 2, \ldots, M) \tag{26}$$

where M is the total number of input variables.

The input and output of the hidden and output layer can be expressed, respectively, as

$$net_k^{(2)}(t_i) = \sum_{j=1}^{M} w_{kj}^{(2)} O_j^{(1)}, O_k^{(2)}(t_i) = f(net_k^{(2)}(t_i)) \;\; (k = 1, 2, \ldots, Q) \tag{27}$$

$$net_l^{(3)}(t_i) = \sum_{j=1}^{Q} w_{lk}^{(3)} O_k^{(2)}(t_i), O_l^{(3)}(t_i) = g(net_l^{(3)}(t_i)) \;\; (l = 1, 2, 3)$$

$$O_1^{(3)}(t_i) = K_p, O_2^{(3)}(t_i) = K_i, O_3^{(3)}(t_i) = K_d \tag{28}$$

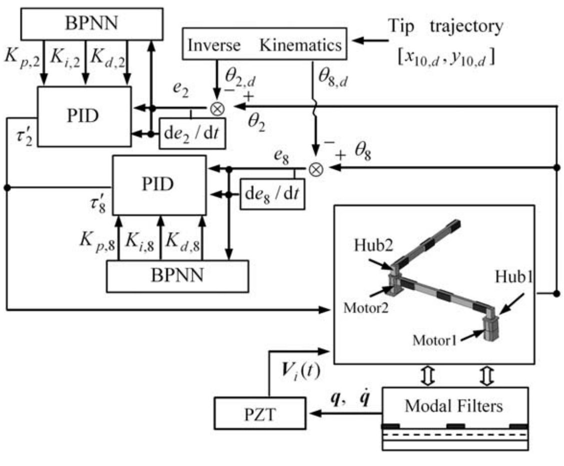

Fig. 5. Control block diagram of the controlled flexible manipulator.

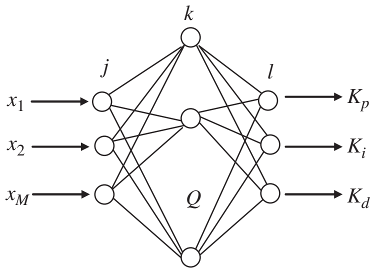

Fig. 6. Structure of BP nerve network.

where $net_k^{(2)}$ and $O_k^{(2)}$ denote the input and output of the hidden layer, $net_l^{(3)}$ and $O_l^{(3)}$ denote the input and output of the output layer, $w_{kj}^{(2)}$ and $w_{lk}^{(3)}$ are the weighting coefficient of the hidden and output layer, the superscripts (1), (2) and (3) represent the input, hidden and output layers, respectively, Q is the total number of hidden layer node.

$$f(net_k^{(2)}(t_i)) = \tanh(net_k^{(2)}(t_i)) = (e^{net_k^{(2)}(t_i)} - e^{-net_k^{(2)}(t_i)})/(e^{net_k^{(2)}(t_i)} + e^{-net_k^{(2)}(t_i)})$$

$$g(net_l^{(3)}(t_i)) = \tfrac{1}{2}(1 + \tanh(net_l^{(3)}(t_i))) = e^{net_l^{(3)}(t_i)}/(e^{net_l^{(3)}(t_i)} + e^{-net_l^{(3)}(t_i)})$$

Set the performance index function as

$$E(t_i) = \tfrac{1}{2}(e(t_i))^2 \tag{29}$$

The weighting coefficients of BPNN are modified by

$$\Delta w_{lk}^{(3)}(t_i) = -\eta \frac{\partial E(t_i)}{\partial w_{lk}^{(3)}} + \alpha \Delta w_{lk}^{(3)}(t_{i-1}) \tag{30}$$

where η is the learning velocity, α is the inertia coefficient.

Then the learning arithmetic of the output layer [36] can be obtained as

$$\Delta w_{lk}^{(3)}(t_i) = \eta \delta_l^{(3)} O_k^{(3)}(t_i) + \alpha \Delta w_{lk}^{(3)}(t_{i-1})$$

$$\delta_l^{(3)} = e(t_i)\mathrm{sgn}\left(\frac{\partial \theta(t_i)}{\partial \Delta\tau(t_i)}\right)\frac{\partial \Delta\tau(t_i)}{\partial O_l^{(3)}(t_i)} g'(net_l^{(3)}(t_i)) \tag{31}$$

Similarly, the learning arithmetic of the hidden layer [36] can be obtained as

$$\Delta w_{kj}^{(2)}(t_i) = \eta \delta_k^{(2)} O_j^{(2)}(k) + \alpha \Delta w_{kj}^{(2)}(t_{i-1}), \delta_k^{(2)} = f'(net_k^{(2)}(t_i)) \sum_{l=1}^{3} \delta_l^{(3)} w_{lk}^{(3)}(t_i) \tag{32}$$

where

$$g'(net_l^{(3)}(t_i)) = g(net_l^{(3)}(t_i))[1 - g(net_l^{(3)}(t_i))]$$

$$f'(net_k^{(2)}(t_i)) = \frac{1 - f^2(net_k^{(2)}(t_i))}{2} f(net_k^{(2)}(t_i))$$

4.2. Design of PZT controller

Based on modal velocity feedback, the command voltage [30–35] applied to PZT actuator i can be obtained as

$$V_i^a(t) = -K_i^a \frac{\partial \dot{u}}{\partial x_2}\Big|_{x_{i,1}^a}^{x_{i,2}^a} \tag{33}$$

where K_i^a is the gain coefficient of PZT actuator i, u is the transverse deformation of arm, $i = 1, 2, 3$.

Using modal method, the transverse deformation u of a beam can be expressed as

$$u(x_2, t) = \sum_{k=1}^{n} Y^k(x_2) q^k(t) \tag{34}$$

where $Y^k(x_2)$ and q^k are the kth generalized eigenvector and generalized coordinate describing the deformation of arm relative to its body-fixed reference system with modal method, the superscript 'n' is the highest order of modal considered, $k = 1, 2, \ldots, n$.

Considering the first three-order modals, the command voltage Eq. (33) can be written as

$$V_i^a(t) = -K_i^a[{}^1s_1 \;\; {}^1s_2 \;\; {}^1s_3]|_{x_{i,1}^a}^{x_{i,2}^a}[\dot{q}^1, \dot{q}^2, \dot{q}^3]^{\mathrm{T}} \tag{35}$$

where ${}^1s_k = \partial Y^k(x_2)/\partial x_2$.

The distributing moment [30–35] produced by PZT actuator i is

$$\overline{m}_i^a(x_2, t) = c_i^a V_i^a(t)[H(x_2 - x_{i,1}^a) - H(x_2 - x_{i,2}^a)] \tag{36}$$

where

$$c_i^a = \frac{1}{2} d_i^{31} E_i^a b_i^a (t_i^a + 2t^b), H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

d_i^{31}, E_i^a, b_i^a and t_i^a are the piezoelectric strain constant, elastic modules, width and thickness of PZT actuator i, respectively, t^b is the thickness of arm.

5. Transfer matrices of controlled flexible manipulator

For the controlled manipulator shown in Fig. 4(a), based on above control strategy, the transfer equation and transfer matrix of each element are developed as follows.

5.1. Transfer matrix of revolute hinge connecting motor and Hub

For the revolute hinge, the position coordinates, orientation angles, interior torques and interior forces of its input end and output end are equal, respectively. From the control Eq. (24) of PID controller, the interior torque of the input end of revolute hinge can be written as

$$m_I(t_i) = (K_p + K_d + K_i)[\theta_O(t_i) - \theta_{O,d}(t_i)] + m_{I,t_{i-1}} - (2K_d + K_p)e_{O,t_{i-1}} + K_d e_{O,t_{i-2}} \tag{37}$$

where e, θ, θ_d have the similar meaning as Eq. (24), subscript O denote the output end of revolute hinge.

From Eq. (37), one can obtain

$$\theta_O(t_i) = u_{34} m_I(t_i) + u_{37} \tag{38}$$

where

$$u_{34} = 1/(K_p + K_d + K_i)$$

$$u_{37} = 1/(K_p + K_d + K_i)[2K_d e_O(t_{i-1}) - K_d e_O(t_{i-2}) + K_p e_O(t_{i-1}) - m_I(t_{i-1})] + \theta_{O,d}(t_i)$$

Define the state vectors of the input end and output end of revolute hinge as

$$\mathbf{z}_I = [x, y, \theta, m, q_x, q_y, 1]^{\mathrm{T}}, \mathbf{z}_O = [x, y, \theta, m, q_x, q_y, 1]^{\mathrm{T}} \tag{39}$$

The transfer equation of revolute hinge can be obtained

$$\mathbf{z}_O = \mathbf{U}\mathbf{z}_I \tag{40}$$

Transfer matrix

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & u_{34} & 0 & 0 & u_{37} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{41}$$

5.2. Transfer matrix of PZT controller

Define the state vectors of the control input and feedback output of PZT controller as

$$\tilde{\mathbf{z}}_f = [\tilde{q}^1, \tilde{q}^2, \tilde{q}^3, 1]^{\mathrm{T}}, \tilde{\mathbf{z}}_C = [V_1^a, V_2^a, V_3^a, 1]^{\mathrm{T}} \tag{42}$$

where $\tilde{q}^1, \tilde{q}^2, \tilde{q}^3$ are the feedback parameters of piezofilm sensors, that is, the first three-order generalized coordinates describing the transverse deformation of arm.

Linearizing Eq. (35), the transfer equation of PZT controller can be obtained

$$\mathbf{z}_C = \mathbf{U}^{\mathrm{fb}}(t_i)\mathbf{z}_f \tag{43}$$

Transfer matrix

$$\mathbf{U}^{\text{fb}} = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u'_1 \\ u_{21} & u_{22} & u_{23} & u'_2 \\ u_{31} & u_{32} & u_{33} & u'_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{44}$$

where

$$u_{ij} = -\chi_3 K_{ai}\,{}^1s_j\,|_{x_{i,1}^a}^{x_{i,2}^a}, u'_i \; = -K_{ai}\,{}^1s_1\,|_{x_{1,1}^a}^{x_{1,2}^a}\chi_{4,q^1} - \cdots - K_{ai}\,{}^1s_n\,|_{x_{i,1}^a}^{x_{i,2}^a}\chi_{4,q^n}, (i = 1,2,3; j = 1,2,3)$$

5.3. Transfer matrix of Euler–Bernoulli moving in plane with PZT control

The dynamics equations of constant section area Euler–Bernoulli beam with PZT control moving in plane as shown in Fig. 7 can be deduced as

$$\begin{aligned} &\dot{\theta}\int_0^l \overline{m}2u\dot{u}\,\mathrm{d}x_2 + \ddot{\theta}\int_0^l \overline{m}(u^2 + x_2^2)\,\mathrm{d}x_2 + \int_0^l \overline{m}x_2\ddot{u}\,\mathrm{d}x_2 + m\ddot{y}_{O_2}x_{O_2C} - m\ddot{x}_{O_2}y_{O_2C} \\ &= \sum_{i=1}^{3}\int_{a_i}\overline{m}_i^a - q_{2,y}(l,t)l - m(0,t) + m(l,t) + \int_0^l x_2 f_{2,y}(x_2,t)\,\mathrm{d}x_2 + \int_0^l m'(x_2,t)\,\mathrm{d}x_2 \end{aligned} \tag{45}$$

$$\frac{\partial q_{2,y}(x_2,t)}{\partial x_2} = f_{2,y}(x_2,t) - \overline{m}(\ddot{u} + x_2\ddot{\theta} - u\dot{\theta}^2 + \ddot{y}_{O_2}\cos\theta - \ddot{x}_{O_2}\sin\theta) \tag{46}$$

$$\frac{\partial q_{2,x}(x_2,t)}{\partial x_2} = f_{2,x}(x_2,t) - \overline{m}(-\ddot{\theta}u - 2\dot{\theta}\dot{u} - \dot{\theta}^2 x_2 + \ddot{y}_{O_2}\sin\theta + \ddot{x}_{O_2}\cos\theta) \tag{47}$$

where $\overline{m}$, l and m are the line mass density, length and mass of the beam, respectively, $\dot{u}$ and $\ddot{u}$ are the first order and the second order derivative of u with respect to time in the body-fixed coordinate system $O_2x_{O_2}y_{O_2}$, x_2 is the coordinate along the beam axis, x_{O_2C}, y_{O_2C} are the projects of the vector from O_2 to mass center of beam onto inertial reference system oxy, x_{O_2}, y_{O_2} are the position coordinates of point O_2 relative to oxy, $q_{2,y}$, $q_{2,x}$, $f_{2,y}$ and $f_{2,x}$ are the interior forces and distributed exterior forces acted on beam in y_2 and x_2 direction, respectively, $m(0,t), m(l,t)$ are interior torques acted on the inboard point and outboard point of beam, m' is the distributed exterior torque acted on beam.

Only considering the first three-order generalized coordinates describing the transverse deformation of arm, let n=3, substituting Eqs. (34)–(47), integrate along the axial direction of beam for Eqs. (46) and (47) and linearizing, then one can obtain

$$q_{2,y}(l,t) = \zeta_{1,1}x_{O_2} + \zeta_{1,2}y_{O_2} + \zeta_{1,3}\theta + q_{2,y}(0,t) + \sum_{k=1}^{3}\zeta_{1,6+k}q^k + \zeta_{1,10} \tag{48}$$

$$q_{2,x}(l,t) = \zeta_{2,1}x_{O_2} + \zeta_{2,2}y_{O_2} + \zeta_{2,3}\theta + q_{2,x}(0,t) + \sum_{k=1}^{3}\zeta_{2,6+k}q^k + \zeta_{2,10} \tag{49}$$

where

$$\zeta_{1,1} = m\overline{s}\chi_1, \zeta_{1,2} = -m\overline{c}\chi_1, \zeta_{1,3} = 2\overline{m}\chi_3\dot{\theta}_{t_{i-1}}\sum_{k=1}^{3}s_k^0 q_{t_{i-1}}^k - \frac{ml}{2}\chi_1$$

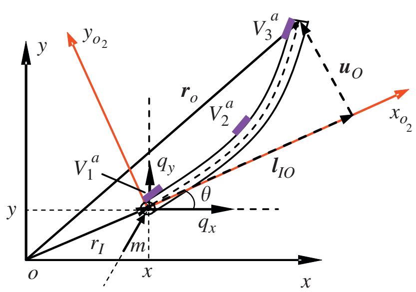

Fig. 7. Beam moving in plane with PZT control.

$$\zeta_{2,1}=-m\overline{c}\chi_1,\zeta_{2,2}=-m\overline{s}\chi_1,\zeta_{2,3}=\overline{m}(l^2\chi_3\dot{\theta}_{t_{i-1}}+\chi_1\sum_{k=1}^{3}s_k^0q_{t_{i-1}}^k+2\chi_1\sum_{k=1}^{3}s_k^0\dot{q}_{t_{i-1}}^k)$$

$$\zeta_{1,6+k}=\overline{m}(\dot{\theta}_{t_{i-1}}^2-\chi_1)s_k^0,\zeta_{2,6+k}=\overline{m}(2\dot{\theta}_{t_{i-1}}\chi_3+\ddot{\theta}_{t_{i-1}})s_k^0(k=1,2,3)$$

$$\zeta_{1,10}=\int_0^l f_{2,y}(x_2,t)\,\mathrm{d}x_2+m\overline{s}\chi_{2,x_{O_2}}-m\overline{c}\chi_{2,y_{O_2}}-\frac{ml}{2}\chi_{2,\theta}+2\overline{m}\chi_{4,\theta}\dot{\theta}_{t_{i-1}}\sum_{k=1}^{3}s_k^0q_{t_{i-1}}^k-2\overline{m}\dot{\theta}_{t_{i-1}}^2\sum_{k=1}^{3}s_k^0q_{t_{i-1}}^k-\overline{m}\sum_{k=1}^{3}s_k^0\chi_{2,q^k}$$

$$\zeta_{2,10}=\int_0^l f_{2,x}(x_2,t)\,\mathrm{d}x_2+2\overline{m}\dot{\theta}_{t_{i-1}}\sum_{k=1}^{3}s_k^0\chi_{4,q^k}-m\overline{s}\chi_{2,y_{O_2}}-2\overline{m}\dot{\theta}_{t_{i-1}}\sum_{k=1}^{3}s_k^0\dot{q}_{t_{i-1}}^k-m\overline{c}\chi_{2,x_{O_2}}+\overline{m}\chi_{2,\theta}\sum_{k=1}^{3}s_k^0q_{t_{i-1}}^k$$
$$+2\overline{m}\chi_{4,\theta}\sum_{k=1}^{3}s_k^0\dot{q}_{t_{i-1}}^k-\overline{m}\ddot{\theta}_{t_{i-1}}\sum_{k=1}^{3}s_k^0q_{t_{i-1}}^k+\frac{ml}{2}(2\chi_{4,\theta}\dot{\theta}_{t_{i-1}}-\dot{\theta}_{t_{i-1}}^2)$$

$$s_k^0=\int_0^l Y^k(x_2)\,\mathrm{d}x_2,s_{i,j}^0=s_{j,i}^0=\int_0^l Y^iY^j\,\mathrm{d}x_2,{}^ms_{k,k'}=\int_0^l {}^mY^k(x_2)Y^{k'}(x_2)\,\mathrm{d}x_2$$

Substituting Eqs. (34), (36) and (48) to Eq. (45) and linearizing, one can obtain

$$m(l,t)=(\xi_1+l\zeta_{1,1})x_{O_2}+(\xi_2+l\zeta_{1,2})y_{O_2}+(\xi_3+l\zeta_{1,3})\theta+m(0,t)+lq_{2,y}(0,t)+\sum_{k=1}^{3}(\xi_{6+k}+l\zeta_{1,6+k})q^k+(\xi_{10}+l\zeta_{1,10})$$
$$-\int_0^l x_2f_{2,y}(x_2,t)\,\mathrm{d}x_2-\int_0^l m'(x_2,t)\,\mathrm{d}x_2-\sum_{i=1}^{3}V_i^a(t)\int_{a_i}c_i^a[H(x_2-x_{i,1}^a)-H(x_2-x_{i,2}^a)\,\mathrm{d}x_2 \quad (50)$$

where

$$\xi_1=-\left(m\overline{c}\sum_{k=1}^{3}Y_c^kq_{t_{i-1}}^k+\frac{l}{2}m\overline{s}\right)\chi_1,\xi_2=\left(\frac{l}{2}m\overline{c}-m\overline{s}\sum_{k=1}^{3}Y_c^kq_{t_{i-1}}^k\right)\chi_1$$

$$\xi_3=\overline{m}\sum_{k=1}^{3}\sum_{j=1}^{3}s_{k,j}^0[2\chi_3q^k(t_{i-1})\dot{q}^j(t_{i-1})+\chi_1q^k(t_{i-1})q^j(t_{i-1})]+m\frac{l^2}{3}\chi_1$$

$$\xi_{6+k}=2\overline{m}\sum_{j=1}^{3}s_{k,j}^0[\dot{\theta}_{t_{i-1}}(\dot{q}_{t_{i-1}}^j+\chi_3q_{t_{i-1}}^j)+\ddot{\theta}_{t_{i-1}}q_{t_{i-1}}^j]+\overline{m}\sum_{k=1}^{3}s_k^1\chi_1-m(\overline{s}Y_c^k\ddot{y}_{O_2,t_{i-1}}+\overline{c}Y_c^k\ddot{x}_{O_2,t_{i-1}})(k=1,2,3)$$

$$\xi_{10}=2\overline{m}\sum_{k=1}^{3}\sum_{j=1}^{3}s_{k,j}^0(\chi_{4,\theta}\dot{q}_{t_{i-1}}^kq_{t_{i-1}}^j+\dot{\theta}_{t_{i-1}}q_{t_{i-1}}^k\chi_{4,q^j}-2\dot{\theta}_{t_{i-1}}q_{t_{i-1}}^k\dot{q}_{t_{i-1}}^j)+\overline{m}\sum_{k=1}^{3}s_k^1\chi_{2,q^k}+\frac{l}{2}m(\overline{c}\chi_{2,y_{O_2}}-\overline{s}\chi_{2,x_{O_2}})+m\frac{l^2}{3}\chi_{2,\theta}$$
$$+\overline{m}\sum_{k=1}^{3}\sum_{j=1}^{3}s_{k,j}^0(\chi_{2,\theta}q_{t_{i-1}}^kq_{t_{i-1}}^j-2\ddot{\theta}_{t_{i-1}}q_{t_{i-1}}^kq_{t_{i-1}}^j)-m\sum_{k=1}^{3}Y_c^k[\overline{s}(q_{t_{i-1}}^k\chi_{2,y_{O_2}}-q_{t_{i-1}}^k\ddot{y}_{O_2,t_{i-1}})+\overline{c}(q_{t_{i-1}}^k\chi_{2,x_{O_2}}-q_{t_{i-1}}^k\ddot{x}_{O_2,t_{i-1}})]$$

As $q_{2,y}(0,t)=-q_x(0,t)\sin\theta+q_y(0,t)\cos\theta$, then Eq. (50) can be written as

$$m(l,t)=u_{4,1}x_{O_2}+u_{4,2}y_{O_2}+u_{4,3}\theta+m(0,t)+u_{4,5}q_x(0,t)+u_{4,6}q_y(0,t)+\sum_{k=1}^{3}u_{4,6+k}q^k+u_{4,10}+\sum_{i=1}^{3}u_{V_i^a}{}'V_i^a \quad (51)$$

where

$$u_{4,j}=l\zeta_{1,j}+\xi_j(j=1,2,3,7,8,9),u_{4,5}=-l\overline{s},u_{4,6}=l\overline{c}$$

$$u_{4,10}=l\zeta_{1,10}+\xi_{10}-\int_0^l x_2f_{2,y}(x_2,t)\,\mathrm{d}x_2-\int_0^l m'(x_2,t)\,\mathrm{d}x_2$$

$$u_{V_i^a}{}'=-\int_{a_i}c_i^a[H(x_2-x_{i,1}^a)-H(x_2-x_{i,2}^a)\,\mathrm{d}x_2(i=1,2,3)$$

Projecting Eqs. (48) and (49) onto inertial reference system, then

$$q_x(l,t)=u_{5,1}x_{O_2}+u_{5,3}\theta+q_x(0,t)+\sum_{k=1}^{3}u_{5,6+k}q^k+u_{5,10} \quad (52)$$

$$q_y(l,t) = u_{6,2}y_{O_2} + u_{6,3}\theta + q_y(0,t) + \sum_{k=1}^{3} u_{6,6+k}q^k + u_{6,10} \tag{53}$$

where

$$u_{5,1} = -m\chi_1, u_{6,2} = -m\chi_1$$

$$u_{5,j} = \bar{c}\zeta_{2,j} - \bar{s}\zeta_{1,j}, u_{6,j} = \bar{s}\zeta_{2,j} + \bar{c}\zeta_{1,j}(j = 3,7,8,9,10)$$

From the geometrical equations, one can obtain

$$\begin{bmatrix} x \\ y \end{bmatrix}_O = \begin{bmatrix} x \\ y \end{bmatrix}_I + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} l \\ u \end{bmatrix}_O \tag{54}$$

Linearizing Eq. (54), then one can obtain

$$x_O = x_I + u_{13}\theta + \sum_{k=1}^{3} u_{1,6+k}q^k + u_{1,10}, y_O = y_I + u_{23}\theta + \sum_{k=1}^{3} u_{2,6+k}q^k + u_{2,10} \tag{55}$$

where

$$u_{13} = -ls - c\sum_{k=1}^{3} Y_O^k q_{t_{i-1}}^k, u_{23} = lc - s\sum_{k=1}^{3} Y_O^k q_{t_{i-1}}^k$$

$$u_{1,6+k} = -sY_O^k, u_{2,6+k} = cY_O^k (k = 1,2,3)$$

$$u_{1,10} = l(c+\theta s) + c\sum_{k=1}^{3} Y_O^k q_{t_{i-1}}^k \theta_{t_{i-1}}, u_{2,10} = l(s-\theta c) + s\sum_{k=1}^{3} Y_O^k q_{t_{i-1}}^k \theta_{t_{i-1}}$$

Define the state vectors of the input end and output end of Euler–Bernoulli beam with PZT control as

$$\begin{cases} \mathbf{z}_I = [x,y,\theta,m,q_x,q_y,q^1,q^2,q^3,1,V_1^a,V_2^a,V_3^a,1]^{\mathrm{T}} \\ \mathbf{z}_O = [\mathbf{z}_{O_1}^{\mathrm{T}}, \tilde{\mathbf{z}}_f^{\mathrm{T}}]^{\mathrm{T}} = [x,y,\theta,m,q_x,q_y,q^1,q^2,q^3,1,\tilde{q}^1,\tilde{q}^2,\tilde{q}^3,1]^{\mathrm{T}} \\ \mathbf{z}_{O_1} = [x,y,\theta,m,q_x,q_y,q^1,q^2,q^3,1]^{\mathrm{T}}, \tilde{\mathbf{z}}_f = [\tilde{q}^1,\tilde{q}^2,\tilde{q}^3,1]^{\mathrm{T}} \end{cases} \tag{56}$$

Considering the generalized coordinates on two ends of beam are equal, then the transfer equation and transfer matrix of the Euler–Bernoulli beam with PZT control can be obtained as

$$\mathbf{z}_O = [\mathbf{z}_{O_1}^{\mathrm{T}} \ \ \tilde{\mathbf{z}}_f^{\mathrm{T}}]^{\mathrm{T}} = \mathbf{U}\mathbf{z}_I = \begin{bmatrix} \mathbf{U}^{O_1} \\ \mathbf{U}^f \end{bmatrix} \mathbf{z}_I \tag{57}$$

where

$$\mathbf{U}^{O_1} = \begin{bmatrix} \mathbf{U}_{11} & \mathbf{O}_{3\times3} & \mathbf{U}_{13} & \mathbf{U}_{15} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{U}_{21} & \mathbf{U}_{22} & \mathbf{U}_{23} & \mathbf{U}_{25} & \mathbf{U}_{24} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} & \mathbf{I}_3 & \mathbf{O}_{3\times1} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & 1 & \mathbf{O}_{1\times3} & 0 \end{bmatrix}, \mathbf{U}^f = \begin{bmatrix} \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} & \mathbf{I}_3 & \mathbf{O}_{3\times1} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & 1 & \mathbf{O}_{1\times3} & 0 \end{bmatrix}$$

$$\mathbf{U}_{11} = \begin{bmatrix} 1 & 0 & u_{1,3} \\ 0 & 1 & u_{2,3} \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{U}_{13} = \begin{bmatrix} u_{1,7} & u_{1,8} & u_{1,9} \\ u_{2,7} & u_{2,8} & u_{2,9} \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{U}_{15} = \begin{bmatrix} u_{1,10} \\ u_{2,10} \\ 0 \end{bmatrix}, \mathbf{U}_{25} = \begin{bmatrix} u_{4,10} \\ u_{5,10} \\ u_{6,10} \end{bmatrix}$$

$$\mathbf{U}_{21} = \begin{bmatrix} u_{4,1} & u_{4,2} & u_{4,3} \\ u_{5,1} & 0 & u_{5,3} \\ 0 & u_{6,2} & u_{6,3} \end{bmatrix}, \mathbf{U}_{22} = \begin{bmatrix} 1 & u_{4,5} & u_{4,6} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{U}_{23} = \begin{bmatrix} u_{4,7} & u_{4,8} & u_{4,9} \\ u_{5,7} & u_{5,8} & u_{5,9} \\ u_{6,7} & u_{6,8} & u_{6,9} \end{bmatrix}, \mathbf{U}_{24} = \begin{bmatrix} u_{V_1^a} & u_{V_2^a} & u_{V_3^a} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{58}$$

5.4. Transfer matrices of fixed hinges connected with beam moving in plane

5.4.1. Fixed hinge whose inboard body is rigid body and outboard body is beam with PZT control

For the fixed hinge whose inboard body is rigid body and outboard body is beam, the position coordinates, orientation angles, interior torques and interior forces of its input end and output end are equal, respectively. The relationship between the generalized coordinates used for describing the deformation of outboard beam and the state vector of its input end is developed as follows.

Combining the control Eq. (36) of PZT actuators, the transverse vibration equation of Euler–Bernoulli beam with PZT control can be obtained as

$$\begin{aligned} & EI\frac{\partial^4 u}{\partial x_2^4} - \rho I\frac{\partial^4 u}{\partial x_2^2 \partial t^2} + \overline{m}(\ddot{u} + x_2\ddot{\theta} - u\dot{\theta}^2 + \ddot{y}_{O_2}\cos\theta - \ddot{x}_{O_2}\sin\theta) \\ & = f_{2,y} - \frac{\partial}{\partial x_2}m'(x_2,t) - \sum_{i=1}^{3} c_i^a V_i^a[\delta(x_2 - x_{i,1}^a) - \delta(x_2 - x_{i,2}^a)] \end{aligned} \tag{59}$$

where E and ρ are the elastic modulus and density of the beam, respectively, I is the inertia product of the cross-section.

Only considering the first three-order generalized coordinates describing the transverse deformation of arm, let n=3, substituting Eq. (34) into Eq. (59), multiplying $Y^{k'}(x_2)$ $(k' = 1,2,3)$ to the two sides of the equation and integrating along x_2, linearizing this equation, using Cramer method, the transfer equation of fixed hinge whose inboard body is rigid body and outboard body is beam with PZT control moving in plane can be obtained.

$$\mathbf{z}_O = \mathbf{U}^{I_1}\mathbf{z}_{I_1} + \mathbf{U}^C\tilde{\mathbf{z}}_C \tag{60}$$

where

$$\mathbf{U}^{I_1} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{3\times3} & \mathbf{I}_3 & \mathbf{O}_{3\times1} \\ \mathbf{U}_{31} & \mathbf{O}_{3\times3} & \mathbf{U}_{34} \\ \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & 1 \\ \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & 1 \end{bmatrix}, \mathbf{U}^C = \begin{bmatrix} \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{U}_{33} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{1\times3} & 0 \\ \mathbf{I}_3 & \mathbf{O}_{3\times1} \\ \mathbf{O}_{1\times3} & 0 \end{bmatrix}, \mathbf{U}_{31} = \begin{bmatrix} \Delta_{11}/\Delta & \Delta_{12}/\Delta & \Delta_{13}/\Delta \\ \Delta_{21}/\Delta & \Delta_{22}/\Delta & \Delta_{23}/\Delta \\ \Delta_{31}/\Delta & \Delta_{32}/\Delta & \Delta_{33}/\Delta \end{bmatrix}$$

$$\mathbf{U}_{33} = \begin{bmatrix} \Delta_{14}/\Delta & \Delta_{15}/\Delta & \Delta_{16}/\Delta \\ \Delta_{24}/\Delta & \Delta_{25}/\Delta & \Delta_{26}/\Delta \\ \Delta_{34}/\Delta & \Delta_{35}/\Delta & \Delta_{36}/\Delta \end{bmatrix}, \mathbf{U}_{34} = \begin{bmatrix} \Delta_{17}/\Delta \\ \Delta_{27}/\Delta \\ \Delta_{37}/\Delta \end{bmatrix}$$

$$\Delta = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}, \Delta_{1j} = \begin{vmatrix} B_{1j} & A_{12} & A_{13} \\ B_{2j} & A_{22} & A_{23} \\ B_{3j} & A_{32} & A_{33} \end{vmatrix}, \ \Delta_{2j} = \begin{vmatrix} A_{11} & B_{1j} & A_{13} \\ A_{21} & B_{2j} & A_{23} \\ A_{31} & B_{3j} & A_{33} \end{vmatrix}, \ \Delta_{3j} = \begin{vmatrix} A_{11} & A_{12} & B_{1j} \\ A_{21} & A_{22} & B_{2j} \\ A_{31} & A_{32} & B_{3j} \end{vmatrix} (j = 1,\ldots,7)$$

$$A_{k',k'} = {}^4s_{k,k'}EI - {}^2s_{k,k'}\rho I\chi_1 + \overline{m}s_{k,k'}\chi_1 - \overline{m}s_{k,k'}\dot{\theta}_{t_{i-1}}^2, A_{k',k} = -\overline{m}s_{k,k'}\dot{\theta}_{t_{i-1}}^2 (k \neq k')$$

$$B_{k',3} = -\overline{m}(s_{k'}^1\chi_1 - 2\chi_3 s_{k',k'} q_{t_{i-1}}^k \dot{\theta}_{t_{i-1}}), B_{k',1} = \overline{m}\overline{s}s_{k'}^0\chi_1, \ \ B_{k',2} = -\overline{m}\overline{c}s_{k'}^0\chi_1, k,k' = 1,2,3$$

$$B_{k',3+i} = -\int_0^l Y^{k'}(x_2)c_i^a[\delta(x_2 - x_{i,1}^a) - \delta(x_2 - x_{i,2}^a)]\,\mathrm{d}x_2 (i = 1,2,3)$$

$$\begin{aligned} B_{k',7} = & \int_0^l Y^{k'}(x_2)f(x_2,t)\,\mathrm{d}x_2 - \int_0^l Y^{k'}(x_2)\frac{\partial}{\partial x_2}m'(x_2,t)\,\mathrm{d}x_2 + \rho I^2 s_{k',k'}\chi_{2,q^{k'}} \\ & - \overline{m}[s_{k',k'}\chi_{2,q^{k'}} - 2q_{t_{i-1}}^{k'} s_{k',k'}(\dot{\theta}_{t_{i-1}}\chi_{4,\theta} - \dot{\theta}_{t_{i-1}}^2) + s_{k'}^1\chi_{2,\theta} + \overline{c}s_{k'}^0\chi_{2,y_{O_2}} - \overline{s}s_{k'}^0\chi_{2,x_{O_2}}] \end{aligned} \tag{61}$$

State vectors

$$\begin{cases} \mathbf{z}_I = [z_{I_1}^{\mathrm{T}}, \tilde{z}_C^{\mathrm{T}}]^{\mathrm{T}} = [x,y,\theta,m,q_x,q_y,1,V_1^a,V_2^a,V_3^a,1]^{\mathrm{T}} \\ \mathbf{z}_O = [x,y,\theta,m,q_x,q_y,q^1,q^2,q^3,1,V_1^a,V_2^a,V_3^a,1]^{\mathrm{T}} \\ \mathbf{z}_{I_1} = [x,y,\theta,m,q_x,q_y,1]^{\mathrm{T}}, \tilde{\mathbf{z}}_C = [V_1^a,V_2^a,V_3^a,1]^{\mathrm{T}} \end{cases}$$

5.4.2. Fixed hinge whose inboard body is beam with PZT control and outboard body is rigid body

For the fixed hinge whose inboard body is beam and outboard body is rigid body, the position coordinates, interior torques and interior forces of its input end and output end are equal, respectively. The orientation angles of its input end and output end satisfy the following relationship:

$$\theta_O = \theta_I + \sum_{k=1}^{n}\frac{\partial Y_I^k(l)}{\partial x_2}q_I^k(t_i) \tag{62}$$

where θ_I and θ_O are the orientation angles of the input end and output end of fixed hinge, Y_I^k and q_I^k are the eigenvectors and generalized coordinates describing deformation of inboard beam relative to the body-fixed reference system with modal method, respectively.

Define the state vectors of the input end and output end of fixed hinge whose inboard body is beam with PZT control and outboard body is rigid body as

$$\mathbf{z}_I = [x, y, \theta, m, q_x, q_y, q^1, q^2, q^3, 1]^{\mathrm{T}}, \mathbf{z}_O = [x, y, \theta, m, q_x, q_y, 1]^{\mathrm{T}}$$

The transfer equation of fixed hinge whose inboard body is beam with PZT control and outboard body is rigid body moving in plane can be obtained.

$$\mathbf{z}_O = \mathbf{U}\mathbf{z}_I \tag{63}$$

where

$$\mathbf{U} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{O}_{3\times3} & \mathbf{U}_{13} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{3\times3} & \mathbf{I}_3 & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & 1 \end{bmatrix}, \mathbf{U}_{13} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \partial Y_I^1(l)/\partial x_2 & \cdots & \partial Y_I^3(l)/\partial x_2 \end{bmatrix} \tag{64}$$

5.5. Overall transfer equation and overall transfer matrix of manipulator system

According to Eqs. (20), (40), (43), (57), (60) and (63), one can obtain

$$\mathbf{z}_{2,3} = \mathbf{U}_2\mathbf{U}_1\mathbf{z}_{1,0}, \mathbf{z}_{4,3} = \mathbf{U}_3^{I_1}\mathbf{z}_{2,3} + \mathbf{U}_3^C\tilde{\mathbf{z}}_{C,4\to3}, \mathbf{z}_{4,5} = \mathbf{U}_4^{O_1}\mathbf{z}_{4,3}$$

$$\mathbf{z}_{8,9} = \mathbf{U}_8\mathbf{U}_7\mathbf{U}_6\mathbf{U}_5\mathbf{z}_{4,5}, \tilde{\mathbf{z}}_{f,4\to3} = \mathbf{U}_4^f\mathbf{z}_{4,3}, \tilde{\mathbf{z}}_{C,4\to3} = \mathbf{U}_{4\to3}^{\mathrm{fb}}\tilde{\mathbf{z}}_{f,4\to3}$$

$$\mathbf{z}_{10,0} = \mathbf{U}_{10}^{O_1}\mathbf{z}_{10,9}, \tilde{\mathbf{z}}_{C,10\to9} = \mathbf{U}_{10\to9}^{\mathrm{fb}}\tilde{\mathbf{z}}_{f,10\to9}$$

$$\mathbf{z}_{10,9} = \mathbf{U}_9^{I_1}\mathbf{z}_{8,9} + \mathbf{U}_9^C\tilde{\mathbf{z}}_{C,10\to9}, \tilde{\mathbf{z}}_{f,10\to9} = \mathbf{U}_{10}^f\mathbf{z}_{10,9} \tag{65}$$

where $\mathbf{U}_1, \mathbf{U}_7$ are the transfer matrices of revolute hinges 1 and 7, defined by Eq. (41); $\mathbf{U}_2, \mathbf{U}_6, \mathbf{U}_8$ are the transfer matrices of rigid body moving in plane, defined by Eq. (21); $\mathbf{U}_4^{O_1}, \mathbf{U}_4^f, \mathbf{U}_{10}^{O_1}, \mathbf{U}_{10}^f$ are the transfer matrices of beam with PZT control moving in plane, defined by Eq. (58); $\mathbf{U}_3^{I_1}, \mathbf{U}_3^C, \mathbf{U}_9^{I_1}, \mathbf{U}_9^C$ are the transfer matrices of fixed hinge whose inboard body is rigid body and outboard body is beam with PZT control moving in plane, defined by Eq. (61); $\mathbf{U}_{4\to3}^{\mathrm{fb}}$, $\mathbf{U}_{10\to9}^{\mathrm{fb}}$ are the transfer matrices of PZT controller, defined by Eq. (44); $\mathbf{U}_5$ is the transfer matrix of fixed hinge whose inboard body is beam with PZT control and outboard body is rigid body moving in plane, defined by Eq. (64).

From Eq. (65), the overall transfer equation and overall transfer matrix of controlled manipulator can be obtained.

$$\mathbf{U}_{\mathrm{all}}[\mathbf{z}_{1,0}^{\mathrm{T}}, \tilde{\mathbf{z}}_{C,4\to3}^{\mathrm{T}}, \tilde{\mathbf{z}}_{C,10\to9}^{\mathrm{T}}, \mathbf{z}_{10,0}^{\mathrm{T}}]^{\mathrm{T}} = 0 \tag{66}$$

where

$$\mathbf{U}_{\mathrm{all}} = \begin{bmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} & \mathbf{U}_{13} & \mathbf{O} \\ \mathbf{U}_{21} & \mathbf{U}_{22} & \mathbf{O} & \mathbf{O} \\ \mathbf{U}_{31} & \mathbf{U}_{32} & \mathbf{U}_{33} & -\mathbf{I} \end{bmatrix}$$

$$\mathbf{U}_{11} = \mathbf{U}_{10\to9}^{\mathrm{fb}}\mathbf{U}_{10}^f\mathbf{U}_9^{I_1}\mathbf{U}_8\mathbf{U}_7\mathbf{U}_6\mathbf{U}_5\mathbf{U}_4^{O_1}\mathbf{U}_3^{I_1}\mathbf{U}_2\mathbf{U}_1, \mathbf{U}_{33} = \mathbf{U}_{10}^{O_1}\mathbf{U}_9^C$$

$$\mathbf{U}_{13} = \mathbf{U}_{10\to9}^{\mathrm{fb}}\mathbf{U}_{10}^f\mathbf{U}_9^C - \mathbf{I}, \mathbf{U}_{31} = \mathbf{U}_{10}^{O_1}\mathbf{U}_9^{I_1}\mathbf{U}_8\mathbf{U}_7\mathbf{U}_6\mathbf{U}_5\mathbf{U}_4^{O_1}\mathbf{U}_3^{I_1}\mathbf{U}_2\mathbf{U}_1$$

$$\mathbf{U}_{21} = \mathbf{U}_{4\to3}^{\mathrm{fb}}\mathbf{U}_4^f\mathbf{U}_3^{I_1}\mathbf{U}_2\mathbf{U}_1, \mathbf{U}_{32} = \mathbf{U}_{10}^{O_1}\mathbf{U}_9^{I_1}\mathbf{U}_8\mathbf{U}_7\mathbf{U}_6\mathbf{U}_5\mathbf{U}_4^{O_1}\mathbf{U}_3^C$$

$$\mathbf{U}_{12} = \mathbf{U}_{10\to9}^{\mathrm{fb}}\mathbf{U}_{10}^f\mathbf{U}_9^{I_1}\mathbf{U}_8\mathbf{U}_7\mathbf{U}_6\mathbf{U}_5\mathbf{U}_4^{O_1}\mathbf{U}_3^C, \mathbf{U}_{22} = \mathbf{U}_{4\to3}^{\mathrm{fb}}\mathbf{U}_4^f\mathbf{U}_3^C - \mathbf{I}$$

The boundary conditions of the controlled system are

$$\mathbf{z}_{1,0} = [0, 0, 0, m, q_x, q_y, 1]^{\mathrm{T}}, \mathbf{z}_{10,0} = [x, y, \theta, 0, 0, 0, q^1, q^2, q^3, 1]^{\mathrm{T}} \tag{67}$$

Applying the boundary conditions and reference input of system, deleting the columns 1, 2, 3, 19, 20 and 21 in $\mathbf{U}_{\mathrm{all}}$, deleting the rows 4, 8 and 18 in $\mathbf{U}_{\mathrm{all}}$, and combining the arithmetic of BPNN introduced in Section 4.1, the unknown quantities in Eq. (66) can be computed. Then, the dynamics of this CMS can be obtained.

Table 1
Structure parameters of flexible manipulator system.

Hub1 (Hub2)	Flexible arms 4 and 10	Motor 2	Piezofilm sensor/PZT
Mass 0.1 kg Gyration radius 0.05 m Moment of inertia 2.08×10^{-5} kg m^2	Bending stiffness 4.13 N m^2 Density 2800 kg m^{-3} Length 1.0 m	Mass 0.15 kg Length 0.14 m	Elastic modulus 6.3×10^{10} N m^{-2} Piezoelectric strain constant 110×10^{-12} C N^{-1} Thickness 0.4 mm Length 0.1 m

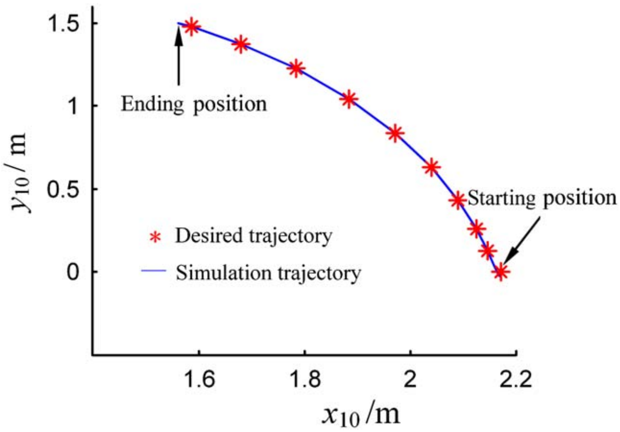

Fig. 8. Tip trajectory of arm 10.

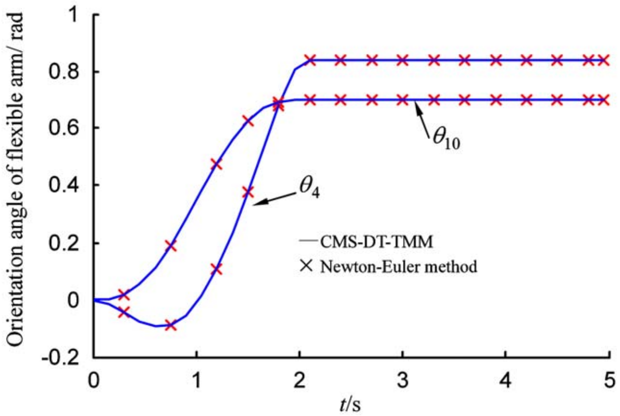

Fig. 9. Time history of orientation angles of arm 4 and 10.

6. Simulation

The structure parameters of the CMS shown in Fig. 4(a) are listed in Table 1. The position of piezofilm sensors/PZT on arm are [0, 0.1 m], [0.45 m, 0.55 m] and [0.9 m, 1 m], respectively. The desired tip trajectory is as follows:

$$x_{10,d}=\begin{cases}1.12\cos\left(\frac{5\pi}{18}t^3-\frac{5\pi}{24}t^4+\frac{\pi}{24}t^5\right)+1.05\cos\left(-\frac{\pi}{4}t^2+\frac{\pi}{3}t^3-\frac{7\pi}{80}t^4\right), & 0\le t\le 2\\ 1.12\cos(2\pi/9)+1.05\cos(4\pi/15), & t>2\end{cases}$$

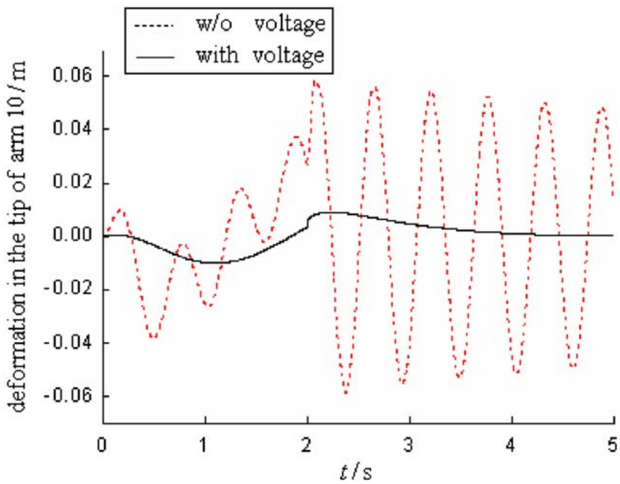

Fig. 10. Time history of transverse deformation in the tip of arm 10.

$$y_{10,d} = \begin{cases} 1.12\sin\left(\frac{5\pi}{18}t^3 - \frac{5\pi}{24}t^4 + \frac{\pi}{24}t^5\right) + 1.05\sin\left(-\frac{\pi}{4}t^2 + \frac{\pi}{3}t^3 - \frac{7\pi}{80}t^4\right), & 0 \le t \le 2 \\ 1.12\sin(2\pi/9) + 1.05\sin(4\pi/15), & t > 2 \end{cases}$$

The dynamics of this CMS are computed by using CMS-dt-tmM and Newton–Euler method, respectively. The tip trajectory of arm 10, the time history of the orientation angles of body-fixed reference system of arms 4 and 10 gotten by the two methods, and the tip transverse deformation of arm 10 with and without PZT control are shown in Figs. 8–10, respectively.

It can be seen from Figs. 8 to 10 that the tip trajectory of arm 10 by simulation has good agreement with its desired trajectory, the tip transverse deformation of arm 10 is restrained speedily under PZT control, and the proposed method and control strategy achieve the tip trajectory tracking and active vibration control of flexible manipulator perfectly. On the other hand, from Fig. 9, we can see the results gotten by the two methods have good agreements, which also validate the proposed method.

7. Conclusions

By taking the control and feedback parameters into account in state vectors, defining new state vectors and deducing new transfer equations and transfer matrices for actuator, controlled element and feedback element, a new method named as CMS-DT-TMM is developed to study dynamics of CMS with real-time control in this paper. Adopting PID adaptive controller and modal velocity feedback control on PZT actuators, applying the proposed method and ordinary dynamics method, respectively, the tip trajectory tracking for a flexible manipulator is carried out. Formulations of the method as well as numerical simulation are given to validate the proposed method.

When using CMS-DT-TMM to study CMS dynamics, the global dynamics equations of system are not needed, and the orders of involved matrices are always very small and irrespective of the size of CMS. This method has the modeling flexibility, higher computational speed, and is efficient for general CMS. Compared with the ordinary dynamics methods, this method has more advantages for dynamic design and real-time control of complex CMS.

This method can be extended to study dynamics of CMS with other control type (feedforward) or other topology structure (branched, close-looped, network, and so on) moving in plane or space, which are yet to be undertaken and will be discussed in detail in another paper.

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